# *k5* Radiometric absolute age calculation < closed system >

Someone told me that each equation I included in the book would halve sales. I therefore resolved not to have any equations at all. In the end, however, I did put in one equation, Einstein's famous equation. I hope that this will not scare off half of my potential readers. —Stephen William Hawking.[1]

The calculation of the absolute age of a specimen from the presence in it of a radioactive-isotope population $N$ depends on the correctness of two hypotheses:

Hypothesis 1

The system to be dated has remained closed so that in it after time $t$ the sum of the surviving parent radioactive-isotope population $N_t$ and the stable-daughter population $D_t$ is equal to the initial-parent population $N_0$.

$$N_t + D_t = N_o$$

Hypothesis 2

The proportional decrease $-dN/N$ of the radioactive-isotope population $N$ at any moment $dt$ is a constant $\lambda$ that is characteristic for the isotope:

$$\frac{-dN/N}{dt} = \lambda .$$

This is a separable equation and can be written as

$$\frac{dN}{N} = -\lambda dt .$$

From this we can obtain the definite integral

namely

$$\ln N \Big|_{N_0}^{N_t} = -\lambda t \Big|_0^t$$

so

$$\ln N_t - \ln N_0 = -\lambda t$$

or

$$\ln \frac{N_t + D_t}{N_t} = \lambda t$$

in which $N_0$ is the number of parent isotopes at the start (time zero) of the decay to be considered, $N_t$ is the number of parent isotopes left, and $D_t$ is the number of end (stable) daughter isotopes accumulated after a lapse of time $t$.

The discovery that individual events are irreducibly random is probably one of the most significant findings of the twentieth century. ... The instant when a radioactive atom decays, or the path taken by a photon behind a half-silvered beam-splitter are objectively random. There is nothing in the Universe that determines the way an individual event will happen. Since individual events may very well have macroscopic consequences, including a specific mutation in our genetic code, the Universe is fundamentally unpredictable and open, not causally closed.
— Anton Zeilinger.[2]

### ***Footnote k4.1*** [3] Modes of decay are by:

**Negative beta emission** occurs when a neutron loses a high energy electron and is thus converted to a proton. This changes the atomic number (number of protons) but not the mass. The decay of Rubidium-87 ( $_{37}^{87}Rb$ ) to Strontium-87 ( $_{38}^{87}Rb$ ) is an example of this type of decay.

**Orbital electron capture** occurs when an electron is incorporated by a neutron in the nucleus. This opposite of negative beta emission results in one less proton and one more neutron. The atomic number (number of protons) changes but not the mass. Potassium-40 ( $_{19}^{40}K$ ) decays to Argon-40 ( $_{18}^{40}Ar$ ) in this manner.

**Alpha particle decay** occurs when the nucleus of an atom emits a particle composed of two protons and two neutrons (which is an alpha particle). This loss changes the atomic mass number (sum of protons and neutrons) and also the atomic number (number of protons). A change in the atomic number produces a new element. An example is the decay of Uranium-238 ( $_{92}^{238}U$ ) to Thorium-234 ( $_{90}^{234}Th$ ) in the uranium decay series.

The half-life $T$ of the radioactive isotope is how long it takes for the radioactive isotope to be reduced by half of its amount. The decay constant $\lambda$ can be expressed in terms of $T$-units by noting that when $t = 1T$, $N_t = N_0/2$. We find that

$$\ln 2 = \lambda\, T\,,$$

$$\lambda = \ln 2 / T\,,$$

or alternatively,

$$T = 0.693... / \lambda\ .$$

The half-life is independent of the number of parent atoms being considered. So, for example, given 100 parent atoms, their number, in the duration of a half-life $T$ will decrease to 50 and the number of stable daughters that come to be is 50. Or beginning with 50 parent atoms, the number of parent atoms will, in the duration of the half-life, decrease to 25 and the number of daughters that come to be is 25. All together, after the duration of two half-lives, 25 parent atoms are left and 75 end daughter atoms have accumulated (***Figure k05.1***).

In practice, the radiometric age of a material is determined not by the use of a graph but by a simple calculation. From hypotheses 1 and 2, the time elapsed $t$ (the radiometric age) in terms of present observables is

$$t = \frac{T}{0.693...} \ln\left(1 + \frac{D_t}{N_t}\right)$$

or, more professionally,

$$t = \frac{1}{\lambda} \ln\left(1 + \frac{D_t}{N_t}\right)$$

This math result, with its smuggled in premise that $\Delta N / \Delta t$ is our $dn / dt$, is final. So, geochronology (the absolute dating of minerals and rocks) is mostly the study of how near to, or far from, the ideal is Hypothesis 1 for samples processed and to justify for each dates obtained. ❑

*Figure k5.1*

The proportion of the parent atoms in a sample declines in time (scaled here in half-life units). The graph is the same for every radioactive parent stable daughter pair when neither has leaked from the sample. Use of the graph is illustrated by the line that has been drawn for the proportion 0.355 of parent atoms left. In this case, the age of the sample is 1.46 x (half-life). If the parent measured is $^{235}U$, then the age of the sample is 1.46 x (0.7) = 1.02 billion years old or if the parent measured is $^{40}K$, then the age of the sample is 1.46 x (1.3) = 1.90 billion years old.